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| TITLE | Structurat imil L | ;uistic Varjables in Problem |
|  | Solving. |  |
| PUB [ATE | 73 |  |
| NOTE | 23 k . |  |

EEFS PRICE DESCRIPTORS

MF-\$C. 65 HC- $\$ 3.29$
Atialysis of Variance; *Arithmetic: Arithmetic Curriculum; *Curriculum Development: *Educational Experiments: Linquage Research; Mathematirs; Models; Prediction; Predictor Variables: Problem Solving; kesearch Methodoloqy: *Statistical Aralysis: Tables (Lata) : Test Construction; Test Selpction

AESTRACI
This paper reports on an experiment desianed to investigate the effect of structural and linguistic varidbles on level of difficulty in solving arithmetic word problems. Identification of such variables is Intended to assist curriculum writers in preparing exercises at a specified level of difficulty tor students at various age levels. The study also considers the variables under varying time conditions and seeks to devise a coding system of the linguistic variables that would improve the accuracy of a linear regression model previously used in aimilar investigations. Details of the theory and methodology of the experiment are provided, and the results are discussed. The signifizance of the structural and lincuistic variables is noted. (VM)

Abstract
Structural and Linguistic Variables
COOUMENI MAS REEN WEDWCS
 in Problem Solving
by
Max Jerman
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Three sections of a methods course for prospective elementary school teachers were siven a set of word problem exercises under different time condit:ons; 20 minutes; one hour; and one day. Structural variables, which accounted for a significant amount of the observed variance in the error rate in arithmetic word problems in Jerman's (1972) study using students in Grades 4-9, were not consistent in entry in a stepwise regression nor did these varisiles account for a significant amount of the total variance. Linguistic variables, however, which were used in Krushinski's (1973) study, showed consistency in early entry and accounted for a reasonable amount of the variance in. the observed proportion correct. The time effect appeared, however, to influence order of entry, more for structural than linguistic variables.

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Previous studies have attempted to identify and define structural and linguistic variables in word problem exercises which account for a large amount of the observed variance in the proportion correct (juppes, Loftus, and Jerman, 1969; Loftus, 1970; Jerman and Rees, 1972; Jerman, 1972a; Jerman, 1972; Krushinski, 1973).

One of the goals of these studies is to identify and quantify. a small set of independent structural variables which will permit curriculum writers to prepare exercises at a specified level of difficulty for students at various age level's. A primary task, in these studies is, therefore, to develop a robust set of variables.

One purpose of this stidy was to determine if a set of structural variables which was found to account for a significant amount of the variance in the observed proportion correct for word-problem exercises solved by students' in grades 4-9 (Jerman, 1972) would alsò account for a significant amount of the observed variance for a different set of word problems solved by college-level students.

A second purpose of this study was to determine whether or not a modification of the coding of the inguistic variables used in the Krushinski (1973) study would improve the accuracy of the linear regression model's prediction.

A third purpose of the study was to test the robustrness of the $r$ variables under varying time conditions.

The six structural variables given in Jerman (1972) and used in this study are defined as follows:

1. Conversion: A count of orie vias assigned if a conversion of units was required to solve the problem, zero otherwise.

Example - If it wa: necessary to change hours to seconds to solve the problem, Conversion $=1$.
2. Recal: The sum of the followirg:
a. One count for a formula to je recalled ald a count for each step in the formula.
b. One count for each conversioin to be recalled and used.
c. One count for each fact fron a previous problem to be recalled and used.

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Example - If the formula \(A=2(1+w)\) was required but no conversions or facts from previous problems were needed,
3 ,
Recal \(=1\) (area formula) \(+2(\) addition and multiplication steps \()+0\) (conversions) \(+0(\) facts \()=3\).
```

3. Length! The number of words in the problem.'
4. Operations 3 (OPER3): The sum of the following:
a. The number of different operations:
b. Four times the number of occurrences of the operation of division.
c. Two times the number of occurgences of the operation of multiplication.
d. The number of occurrences of the operation of addition. Example - If the solution wa: of the form. $([16 \times 30) \times 40] \div 3+1)$,

OPER $3=3$ (3 different operations) $+1 \times 4$ (1 occurrence of division)
$+2 \times 2$ (2 occurrences of muitiplication) +1 (1 occurrencé of
addition) $=12$
5. NOMC2: A count of 1 was assigned for each time a regrouping occurred in each multiplication exercise in the protblem ${ }^{2}$.

Example - ${ }^{14} 38$
$\frac{25}{190}$ NOMC2 $=3$
76
Regrouping's - 1. $5 \times 8=40$ units $=0$ units +4 tens
2. $5 \times 3$ tens +4 tens $=19$ tens $=1$ hundred $\pm 9$ teris
3. 2 tens $\times 8=16$ tens $=1$ rundred +6 tens
6. Qu,tient (QUO): A count of 1 was given for each digit in the quotient if division was requiired and zero otherwise ${ }^{3}$.

$$
\text { Example - } 22 / \frac{303}{6666} . \quad Q U 0=3
$$

The 14 linguistic variables, given by krushinski (1973) and derived
from Golub (1971), used in the study are defined às follows:

1. Number of words (HWO): The number of words in the problem ${ }^{4}$.
2. Nurnber of Sentences (NSEIS): The number of senteaces in the proviem.
3. Sentence qength(SENLTH): The ratio of NWD to NSEII.

- A whole number wals given a count of 1.
${ }^{2}$ Regrouping in summing to obtain the product was not counted.
${ }^{3}$ If the quotient did not terminate after 2 decimal places, a count of 3 was given, for the decimal places ( 2 places and round off).

$$
\begin{aligned}
\text { Examples }- & \frac{1.42}{9.94} \\
\cdots \frac{1.42}{10} & \text { QUO }=13 \\
& \text { QUO }=4
\end{aligned}
$$

${ }^{4}$ A numeral was given a count equal to the number of words in the expression of the numeral in words. Example - 1226 - one thousand two huadred twenty-six (count = 5).
4. Number of Main Clauses (: ) : The number of main clauses in the problem.
5. Number of Subordinate Clau:S (HSC): The number of subordinate clauses in the problem.
6. Number of Clauses (NC): The sum of NMC and.NSC.
7. Number of Words in the Main Clauses (WDMC):

The number of w ids in the main clauses (identified by NMC) in the problem.
8. Number of Woras in the Subordinate Clauses (WDSC): The number of words in the subordinate clauses :.... ified by NSC ) in the problem.
9. Clause Length (CLTH): The ratio of NWO to NC.
10. Main Clause Length (MCLTH): The ratio $0^{\circ}$. $C$ to NMC.
11. Subordinate Clause Length (SCLTH): The ratio of WOSC to HSC.
12. $\frac{\text { llumber of }}{1} \frac{\text { Prorositional }}{\text { Phrases (HPP): The number of propositional }}$ phrases in the problem.
13. Number of Words in the Question Sentence (NWQS): The number of words in.the question sentence.
14. Numerals in the Question Sentence (NUMQS): Assigned a value of one if the question sentence contained a numeral and a value of zero otherwise:

## Methods

## Subjects

Seventy-six prospective elementary school teachers enrolled at Pennsylvania State University in three sections of Elementary Education 326, Teaching Elementary School ${ }^{\text {mithmetic, participated } \text { in the study. Mpst }}$ had taken onty algebra and geometry y n high school and a prerequisite course, Mathematics 200-Number Systems, at Peṇnsylvania State University. Sections 1,11, and 111 had 29, 27, and 20.studepts enrolled, respectively. Students registered for the course according. to how it would fit into their schedule. Scheduling was done by a computer:

Problem Set
The Mathematics Aptitude Test (MAT-R2), published by the Educational Testing Service; was the problem set used in the study. This set consists of word problems requiring arithmetic or very simple algebraic concepts only. The test, which contains two parts, each consisting of 15 items with a time limit of 10 minutes per part, is suitable for students in grades 11-16.

The probtems without answer choices were mimeographed, five to a page, with work' space provided for each problem. Each lab instructor administered the problem set at a time convenient to the schedule of his class. The' test was administered to sections 1 and 111 in the middle of April, 1972 , and to Section 11 at the beginning of May, 1972. Students in Section 1 had 20 minutes during a lab session to complete the exercises, students in Section II had I hour during a lab period to comlete the problems, and students in Section 111 were allowed one day to de the problems at home. Students.in Section lll were asked not to collaborate or seek help.in doing the exercises.

The authors scored each problem for each student as either correct (correct answer), incorrect (an attempt of somé kind was made but there was not a correct answer given) or omit (no attempt was made in solving the probleml. Problems which were omitted were not included in the analysis. The MAT-R2 rest was coded first for the above set of six structural variables and then for the above set of 14 linguistic variables.

A stepwise linear regression program, $B M D O 2 R$ (UCLA), which was modified to include a log transform; and an antilog transform to produce prob'abilities between 0 and 1 , was appliéd to the MAT-R2 test. The regression program was applied for the set of six structural variables alone, the set of 14 linguistic variables alone, and the set of 20 combined variables to each of the three sections and to the three sections, combined into a single group. In addition, an item analysis program, ITEMPGMI (Stanford), and an analysis of variance program, ANOVR (Pennsylvania State University) were applied to the MAT-R2 test.

## Results

The mean total score on the test was 13.00 , with a standard. deviation of 4.945. Cronbach's alpha was .824 , with error of measurement 2.074. The range of scores was 24 , from a low of zergeorrect to a high of 24 correct.

The means and standard devipatig/s for each of the three sections, using for each student both the observed proportion correct of the 30 problems, disregarding omits, and the number of correct responsès, are given in Table 1.

Insert Table 1 About Mere

A summary of the one-way analysis of variance procedule, using both the observed proportion correct and the number of correct responses is given in Table ${ }^{\text { }}$

Insert Table 2 About Here

The calculated fof 4.204, obta: 'red by using the observed, proportion correct, was significant at the 05 level: Using the number of correct responses produced a calsulated $F$ of 20.243 wivich was sighificant at the .001 level. Hence, the sample data do not support, in either application of the analysis of variance, the hypothesis that the three population means were equal. It seems clear, from the data in Table 1 , that there was a: diffet relationship between time allowed students to solve the problems and the number of correct solutions per student. The mean of the number of correct responses for students in section 111 , the one day section, was almost 80 percent greater than the mean of the number of correct responses for students in section 1 , the 20 minute section. The number of correct $/$ responses gives a better indication of the time effect on difficulty-level than does the observed proportion correct since the latter dipregards omits, thus not giving a clear indication of problem difficulty if a lange number of people omit the problem.

- The order of entry of the variables, $R$, and $R^{2}$ for each of the first six steps in the regression for each of, the three sections and for the three sections combined are given in Tatles 3,4 , and 5 for the six structural, 14 linguistic, and 20 combined variables, respectively. In addition, $R$ and $R^{2}$ for the last step are also given. . . i.

[^0]As shown in Table 3 , for the three sections combined, $R_{6}=.64$, with $R_{6}^{2}=.41$. Thus these six structural variables accounted for only 41 percent of the variance in the observed proportion of correct responses. The order of entry of the variables in section 1 , the section with a 20 minute time limit, was decidedly different from that of sections 11 and III. However, the orders of entry, of the first five variables that entered 'the regression for sections 11 and 111 were identical except that second and third variables were interchanged.

The regression $\quad$ coefficients, standard errors of regression coefficients, and computed $t$-values for each of the three sections' and for the tree e sections combined are presented in Tables 6,7 , and 8 for the six structural. 14 linguistic, and 20 combined variables, respectively.

Insert Tables $6 \cdot 7 .$, and 8 About Here

As indicated in Table 6, the structural -variables, which entered the regression, were almost never significant for any group tested.
 proportion correct was accounted for by these six variables. It is interesting that three variables NSEN, NUMQS, and NC entered in the fir. st six steps. in each " the three sections. Especially noteworthy is that in addition to these three variables, CLTH and NPP entered in the first sid steps in. both sections 1 and 111 . Hence, the five variables, NSEN, NUMQS, iC, CLTH, and NPP entered in the first ? ${ }^{\text {良 steps in both section } 1 \text {, the } 20 \text { minute }}$
section and section 111 , the one day section.
As indicated in Table 7, the first four variables, which entered the regression for the three sections combired were significant. Although the six variables ich entered the regression in sectipn 1 and the first four which entered if section lll were significant, none of the first six variables were significant for section 11 , the one hour section.

Especially significant is that the multiple $R$ at step six for section 11:, the one day section, wàs $R=.86$, with $R_{6}^{2}=.74$ : The multiple $R$ atp the last step was $R_{L}=.96$ with $R_{L}^{2}=.91$.

In the 29 variable combined analysis, as show in Table 5, two of the ' 1
first five variables which entered the regression for the tnree sections combined were structura: variables: OPER 3 , the operations variable, and Quo, the division variable. The remaining foun variables which entered in the first six steps were the inguistic variables NC, NUMQS, NPP, and MCLTH, The regression proouced $R_{6}=.83, R_{6}^{2}=.67$, and $R_{L}=.96$, with. $R_{L}^{2}=.95$. Of these six variables, as is shown in Table 8, the significant variables were three linguistic variables, MCLTH, NPP, and NC, and the structural variable OPER3. CLTH and $N C$ entered in the first six steps for each of the three sections, with NC entering first in both ser tions 11 and lll. These two variables were significant each time they occurred except that CLTH was not a significant variable for section ll. Zero, two, and

There was a substantial time effect with respect to the mean number of correct responses for the three sections. This effect, howeve - seemed to be different for the structural and lingujstic variables.

There was almost no consistency in the order of entry of the six structural variables in section 1 , the 20 -minute section, and section $1: 1$, the one day ection. In tact, OPER 3, which entered first in the regression for section 1 , entered last for section 111 and QuO which entered last in section 1 enterec second in section lll.

Linguistic variables, however, were consistent with respect to early entry in the regression analysis for sections 1 anc 111 . Of the fourteen linguistic variables used, five variables, lUMMS, $\operatorname{IPP}$, CLTH, NC, and. MSEN entered in the first six steps for, both sections. In. addition, these five variables eftered in the first five steps in the linguistic regression for the three sections combined. Four of these five variables, NUMQS, NPP, CLTil, and NC were signifisant variables for section 1 , section III,
$r$ and the three-sections combined. No structural variables, however, were significant variables for any of these groups in the structural regression analysis. Three linguistir variables, NSEN, NC, and NUMQS entered in the first six steps fer each of the three sections: Very possibiy the time effert. is not as important with linguistic variables as it is with structural variables.

Therewas a significant difference in the amount of variance accounted for at step six in the structural and linguistic analysis. Disappointingly, the 'regression, for the three sections combined, using the six structural variables, produced $R_{6}^{2}=.41$. thus accounting for, at step 6 : only 41 percent of the total variance in the observed proportion correct. The maximum value.
obtained for $R_{6}^{2}$ in any structural regression was . 47, which occurred for section lll. Hence, even with a full day time limit, the six structural variables alone accounted for only 47 percent of the variance in the observed proportion correct.

In the study by Jerman (1972l of students in grades $4=9$, the first:"
 and NOMC2, produced $R_{3}^{2}=.47$, thus, accounting for 47 percent of the total variance in just three steps. Possibly,(structural variables are not as good predictors of difficulty leved ir arithmetic word problems for prospective teachers as for elementary or junior high schogi children.

The linguistic variables acgounted for a greater amouņt of the total variance than did the structural variables. The nultiple $R$ at step six for section. 111 , the one day section, for which the time effect was insignificant, was .86 with $R_{6}^{2}=.75$. This is a reasonably high percent of variance accounted for at step.six and isquch larger than the $\sim_{R}^{2}$ produced at step six, $<$ in any gegression anatysis using the six structural variables alone. Pérhaps linguistic variables are bettér predictors, than structural variables in arithmetic word problems for the population in question.

An analysis of the 20 combined variables also indicates ithe posible importance of the linguistic variables. Although two structural variables, OPER3 and QUO, entered the regression in the first six, steps for the three sections combined, only OPER 3 was significant whereas three of the linguistic variables which entered in the first six steps, MCLTH, NPP, and NC were significant. Only two structural variables, NOMC2, which was sjgnificant the one time it occurred, and QUO, which was not significant, entered in the first six steps for any of the three sections in the combined

20 variable analysis. In fact, only zero, two, and one structural variables entered the regression in the fïrst six steps for sections 1, 11, and 111, respectively. Two linguistic variables, however, NC and CLTH, entered in the first six steps for each of the three sections. In addition, -
although there was no siructural variable which entered the regression analysis, for the 20 combined variables, in the first six steps for both sections 1 and $1!1$, fqur linguistic variables, NUMCS, NC, CLTH, and iIPP, did.

Perhaps the importance of the linguistic variatles may best be seen by o'servation of the results of the 20 combined variables for-section 111 , the one day section. QUO was the only structural variable among, the first six variables which entered the regression. Although QUO. was rot significant, the five linguistic variables which entered in the first six steps were all significant at the . 005 level, except NWOQS , which was significànt at the . 025 level.

This study should be looked upon only as investigatory in nature. Future studies must be better controlled with respect to uniformity in administration problem sets, ágreater ratio of problems to variables should be included if at all practical, the order of problems should be randomly assigned to students in order to minimize the interaction effect of problem order and difficulty level, and possitly; if enough time is given so that students have ample opportunity to try each problem, the observed proportion correct could be redefiried to be the ratio of correct responses to total number of students so that omitting a problem would reflect not being able to do it and not simply the lack of sufficient time to attempt the problem.

The results of this study are encouraging but not satisfactory. The percent of variance accounted for at step six, in any regression, was not sufficiently large. In addition, there obviously is a great deal of dependency between the variables, especially the linguistic variables. Hence, a refinement and further development of these variables is needed. A basic objective for future study is to produce a set of five or six independent variables which account for a significant amount of the variance in the difficulty level of the arithmetic word-problems and which possess the following characteristics:

1. Each variable permits unambiguous, unique, quantitative coding.
2. Each variable permits easy coding; i.e. it will not be difficult fortancrained person to apply the definition of the variable to the coding of problems.
3. Each variable is comprehens,ive in its applicability to elementary arithmetic word-problems..
4. There is a direct, although not necessarily linear, relationship beitween the numerical value of the variable and the difficulty level of the problem.

Given that such a set of variables could be constructed, then problems with a specified difficulty level could be written so that curriculum developers and teachers would be in a much better position to control the difficulty level of arithmetic word-problems when preparing instructional

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Means and Standard Deviations for Each Section Osing Observed Proportion Correct and Number of Correct Responses

For Each Student

| Section | Observed, Proportion Correct |  | Number of Correct Pesponses |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\bar{x}$ | S | $\overline{\mathrm{x}}$ | S |
| 1 | . 546 | . 1999 | 10.207 | 4.126 |
| 11 | . 654 | . 1405 | 13.148 | 4.148 |
| 111 | . 662 | . 1291 | 17.950 | 4.334 |


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TABLE 4
Order of Entry, $R$, and $R^{2}$. For First Six Steps
$\cdots R_{L}$, and $R_{L}^{2}$, for 14 Linguistid Variables

| Step | 1-20 Minutes |  |  | 11-1 Hour |  |  | 111-1 Day |  |  | Combined |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Variable | R | $\mathrm{R}^{2}$ | Variable | R | $\mathrm{R}^{2}$ | Variable | R' | $\mathrm{R}^{2}$ | Variable | R | $\mathrm{R}^{2}$ |
| 1 | - NMC | . 4524 | . 2046 | HC | . 4940 | . 2441 | NC | . 5685 | . 3231 | NC | . 5433 | . 2952 |
| 2 | NSEN | . 5667 | . 3212 | 'numbs | . 5422 | . 2940 | CLTH | . 6887 | .4742 | NUMQS | .6158 | . 3793 |
| 3 | NUMQS | . 6086 | . 3704 | SENLTH | . 6070 | . 3684 | HPP | . 7441 | . 5537 | CLTH | . 6529 | . 4263 |
| 4 | TJC | . 6414 | . 4114 | HWOQS | .6371 | . 4059 | numas | . 7920 | . 6273 | NPP | . 7506 | . 5634 |
| 5 | CLTH | . 6710 | . 4502. | MCLTH | . 6532 | . 4267 | NWÓQS | . 8508 | . 7238 | NSEN | . 7866 | . 6187 |
| 6 | NPP | . 7336 | . 5382 | NSER | . 6665 | . 4443 | USEN | . 8618 | . 7428 | WDSC | . 7915 | . 6265 |
| Last |  | . 78 | . 61 |  | . $7!$ |  |  |  | . 83 |  |  | . 71 |

table 5
Order of Entry, $R$, and $R^{2}$ For first Six Steps


| Step | 1-20.Minutes |  |  | 11-1 Hour |  |  | 111-1 Day |  |  | Combined |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Variabl | R | $\mathrm{R}^{2}$ | Variable | R | $\mathrm{R}^{2}$ | Variable | R | $\mathrm{R}^{2}$ | Variable | - R | $R^{2}$. |
| $!$ | NMC, | . 4524 | . 2046 | HC | . 4940 | . 2441 | NC | . 5685 | . 3231 | OPER 3 | . 5467 | . 2988 |
| 2 | HSEN | . 5667 | . 3212 | HOMC2 | . 6487 | . 4208 | Quo - | . 7082 | . 5051 | NC | . 7037 | .4952 |
| 3 | numbs | . 6086 | . 3:04 | Quo | . 7009 | . 4912 | CLTH | . 7500 | . 5625 | NUMQS | . 7227 | . 5223 |
| 4 | NC | . 6414 | . 4114 | MCLTH | . 7226 | .5222 | NPP | . 7843 | .6151 | QUO | . 7436 | . 5529 |
| 5 | CLTH | .6710 | . 4502 | CLTH | . 7478 | . 5592 | NUMQS | . 8232 | . 6776 | HPP | . 7638 | . 5833 |
| 6 | NPP | . 7336 | .5382 | HSEN | . 7557 | . 5711 | NWORS | . 8647 | .7477 | MCLTH | . $8277^{\circ}$ | . 6851 |
| Las |  | . 82 | . 87 |  |  | . 76 |  |  | $.91{ }^{\circ}$ |  |  | . 93 |

TABLE 6
Regression Coefficients, Standard Errors of Regression Coefficients, and Computed t-Values For Six Structural Variables

TABLE 7
Regression Coefficients, Standard Errors of Regression Coefficients, and Computed t-Values for 14 Linguistic Variables

TABLE 8
Regression Coefficients, Standard Errofs of Regression Coefficients, and Computed :-Values For 20 Combined Variables



[^0]:    Insert Tables 3,4, and 5 About Here

